**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. **0.2676 - Ans**
4. 0.5
5. 0.6987
6. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
7. More employees at the processing center are older than 44 than between 38 and 44. **False**

Mean = 38

SD = 6

Z score = (Value - Mean)/SD

Z score for 44  = (44 - 38)/6  = 1  i.e.,  84.13 %

Probability of employees above 44 age = 100 - 84.13 =  15.87%  i.e.,  63 out of 400 employees

Z score for 38  = (38 - 38)/6 = 0, from Z table Probability is 50%

Hence % employees between 38 & 44  age = 84.13 - 50 = 34.13 % i.e., 137 out of 400 employees

**Hence more employees at the processing center are between 38 and 44 than older than 44.**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees. **True**

Z score for 30  = (30 - 38)/6 =  -1.33,

from Z table Probability is 9.15  %   i.e., 36 out of 400

**Hence a training program for employees under the age of 30 at the center would be expected to attract about 36 employees.**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Let's consider two independent and identically distributed normal random variables X1​ ~ N(μ, σ2) and X2 ~ N(μ, σ2). According to the Central Limit Theorem, any large sum of independent, identically distributed random variables is approximately Normal.

**1. Distribution and Parameters of 2X1:**

* If X1 follows a normal distribution with mean μ and variance σ2 then 2 X1 will also be a normal random variable. The mean of 2 X1 is 2μ and the variance is 4σ2.
* Hence, 2 X1 ​~ N(2μ, 4σ2)

**2. Distribution and Parameters of X1 + X2:**

* The sum of two independent normal random variables is also normally distributed. If X1 and X2 are independent and identically distributed, then X1 + X2 follows a normal distribution.
* The mean of X1 + X2 is the sum of the means of X1 and X2, so it is 2μ. The variance of X1 + X2 is the sum of the variances of X1 and X2, so it is 2σ2.
* Hence, X1 + X2 ​~ N(2μ, 2σ2)

**In summary:**

* 2 X1 follows a normal distribution with mean **2μ**, and the variance is **4σ2** and X1 + X2 follows a normal distribution with mean **2μ**, and the variance is **2σ2**.
* These results show how the mean and variance of the resulting distributions are impacted by specific linear transformations (scaling and summing) of normal random variables. The linear transformation affects the resulting distribution's mean in both scenarios, but it has a different effect on the variance for summing and scaling operations.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. **48.5, 151.5 Ans**
6. 90.1, 109.9

P (a < X < b) = 0.99, μ = 100, σ = 20

Finding symmetric values for the standard normal distribution such that the area enclosed is 0.99. We must exclude area of .005 in each of the left and right tails. Hence, we want to find the 0.5 and the 99.5 percentiles Z score values.

Z value at 0.5 percentile is given as Z(0.5) = -2.576

Z value at 99.5 percentile is given as Z(99.5) = 2.576

Z = (x - 100)/20 = > x = 20 z + 100

a = -(20 \* 2.576) + 100 = 48.5 and b = (20 \* 2.576) + 100 = 151.5

**Hence the values of a & b symmetric about mean are 48.5, 151.5.**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

M1 = 5, M2 = 7, σ1= 3, σ2= 4

**Combined mean profit of both divisions.**

Mean Total = M1 + M2 = 5 + 7 = 12 million $

Mean Total (in Rs) = (Total Mean \* 45) / 10 = (12 \* 45) / 10 = **54 Crore Rs.**

**Combined Standard Deviation of both divisions.**

Combined Std = (σ12 + σ22)^1/2 = (32 + 42)^1/2 = 5 million $

Combined Std (in Rs) = (Combined Std \* 45) / 10 = (5 \* 45) / 10 = **22.5 Crore Rs**

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

From Python code

stats.norm.interval(0.95, Total Mean, Combined Std)

**The Annual Profit of the Company ranges between [9.9 to 98.1] Crore Rupees 95% of the time.**

1. Specify the 5th percentile of profit (in Rupees) for the company.

From python code

Z = stats.norm.ppf(0.05)

Fifth percentile = (Z \* Combined Std) + Mean Total

**The 5th Percentile of profit for the company is 17 Crore Rupees.**

1. Which of the two divisions has a larger probability of making a loss in the given year?

From python code

division\_1 = (stats.norm.cdf(0,5,3))\*100

division\_2 = (stats.norm.cdf(0,7,4))\*100

The Probability of Division 1 making a loss is 4.78 %

The Probability of Division 2 making a loss is 4.01 %

**The Division 1 has a larger Probability of making a loss.**